# Logarithmic Functions and Their Graphs (2.11) Homework

1. Given $f(x)$ is a logarithmic function, find the value of $k$ in the following table.

| $x$ | 17 | 14 | 8 | $-4$ | $k$ | $-76$ |
|---|---|---|---|---|---|---|
| $f(x)$ | $-8$ | $-6$ | $-4$ | $-2$ | 0 | 2 |

2. Let $f(x) = 3\log_2(x-1) + 5$
   a. Find the domain and range of $f(x)$.
   b. Find the domain and range of $f^{-1}(x)$.
   c. Is $f(x)$ an increasing function or a decreasing function over its domain?
   d. Is $f(x)$ concave up or concave down over its domain?
   e. Determine the end behavior of $f$ as $x$ increases without bound. Express your answer using the mathematical notation of a limit.

3. Let $g(x) = -\log_2(2x-6) - 4$
   a. Find the domain and range of $g(x)$.
   b. Find the domain and range of $g^{-1}(x)$.
   c. Is the rate of change of $g(x)$ increasing or decreasing over its domain?
   d. Is the rate of change of $g(x)$ positive or negative over its domain?
   e. Determine the end behavior of $f$ as $x$ increases without bound. Express your answer using the mathematical notation of a limit.

4. Let $h(x) = 5\log_2(-x) + 1$
   a. Find the domain and range of $h(x)$.
   b. Find the domain and range of $h^{-1}(x)$.
   c. Determine the end behavior of $f$ as $x$ decreases without bound. Express your answer using the mathematical notation of a limit.
   d. Which one of the following describes $h(x)$?
      (A) Increasing at an increasing rate
      (B) Decreasing at an increasing rate
      (C) Increasing at a decreasing rate
      (D) Decreasing at a decreasing rate

5. Match each function below with the letter corresponding to the correct limits describing the function's vertical asymptote and end behavior.

   $f(x) = \ln(x)$
   $g(x) = \ln(-x)$
   $h(x) = -\ln(x)$
   $k(x) = -\ln(-x)$

   (A) $\lim\limits_{x\to 0^+} f(x) = \infty$ and $\lim\limits_{x\to\infty} f(x) = -\infty$
   (B) $\lim\limits_{x\to 0^+} f(x) = -\infty$ and $\lim\limits_{x\to\infty} f(x) = \infty$
   (C) $\lim\limits_{x\to 0^-} f(x) = \infty$ and $\lim\limits_{x\to-\infty} f(x) = -\infty$
   (D) $\lim\limits_{x\to 0^-} f(x) = -\infty$ and $\lim\limits_{x\to-\infty} f(x) = \infty$

6. Let $g(x) = -2\log_4(x+1)$. Which one of the following describes the vertical asymptote of $g(x)$?
   (A) $\lim\limits_{x\to -1^-} g(x) = \infty$
   (B) $\lim\limits_{x\to -1^-} g(x) = -\infty$
   (C) $\lim\limits_{x\to -1^+} g(x) = \infty$
   (D) $\lim\limits_{x\to -1^+} g(x) = -\infty$

7. Find all points of inflection on the graph of $g(x) = \log_3(81x)$.

8. $h(x)$ is of the form $h(x) = a + b\log x$. The graph of $y = h(x)$ contains the points $(0.1, -9)$ and $(100, 9)$.
   A.
      i. Write two equations that could be used to find the values for constants $a$ and $b$.
      ii. Find the values of $a$ and $b$ as decimal approximations.
   B.
      i. Find the average rate of change of $h$ from $x = 0.1$ to $x = 100$. Express your answer as a decimal approximation. Show the computations that lead to your answer.
      ii. Use the average rate of change found in (c) to estimate $h(10)$. Show the work that leads to your answer.
      iii. Using the shape of the graph of $y = h(x)$, explain why the approximation obtained in part B(i) underestimates the actual value of $h(10)$.

9. Find the $x$-value corresponding with each local maximum of the function $f(x) = 2\log_4(x - 5) + 7$.

10. $h(x)$ is a logarithmic function with a few values given in the following table.

| $x$ | $-18$ | $-2$ | $22$ | $58$ | $112$ |
|---|---|---|---|---|---|
| $f(x)$ | $-3$ | $0$ | $3$ | $6$ | $9$ |

   a. Is $f(x)$ increasing or decreasing?
   b. Is $f(x)$ concave up or concave down?
   c. Find $f^{-1}(12)$

11. Consider the function $f(x) = 3\ln(x + 4)$. Which is greater, the average rate of change of $f(x)$ over the interval $[0,3]$ or the average rate of change of $f(x)$ over the interval $[3,6]$? This is not a calculator question.

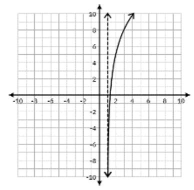
12. Let $g(x) = \log_3 x$.
   a. Find $g^{-1}(2)$
   b. Find $g^{-1}(-1)$
   c. Applying a horizontal dilation by a factor of $\frac{1}{3}$ to this function is equivalent to a vertical translation by $k$ units. Find $k$.
   d. The graph of $y = g(\sqrt{x})$ is the image of the graph of $g(x)$ after a vertical dilation by a factor of $c$. Find $c$.

13. Let $f(x) = \log_2(x)$ and let $g(x) = \log_{\frac{1}{4}}(x)$. Which one of the following accurately describes the transformation that maps the graph of $f(x)$ to the graph of $g(x)$.
   (A) The graph of $g(x)$ is the image of the graph of $f(x)$ after a vertical dilation by a factor of $\frac{1}{8}$ and a reflection over the $y$-axis.
   (B) The graph of $g(x)$ is the image of the graph of $f(x)$ after a vertical dilation by a factor of 8 and a reflection over the $y$-axis.
   (C) The graph of $g(x)$ is the image of the graph of $f(x)$ after a vertical dilation by a factor of 2 and a reflection over the $x$-axis.
   (D) The graph of $g(x)$ is the image of the graph of $f(x)$ after a vertical dilation by a factor of $\frac{1}{2}$ and a reflection over the $x$-axis.

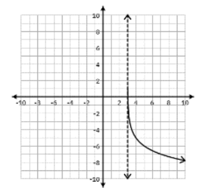
14. The function $f(x)$ is given by $f(x) = 3\log_3(x) - 2\log_9(x)$. Rewrite $f(x)$ as a single logarithm base 3 without negative exponents in any part of the expression. Your result should be of the form $\log_3$(expression).

# Logarithmic Functions and Their Graphs HW Answers

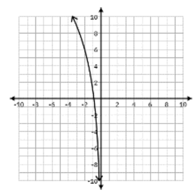
1. $k = -28$
2. 
   a. Domain: $(1, \infty)$ Range: $(\infty, \infty)$
   b. Domain: $(-\infty, \infty)$ Range: $(1, \infty)$
   c. Increasing
   d. Concave Down
   e. $\lim\limits_{x \to \infty} f(x) = \infty$



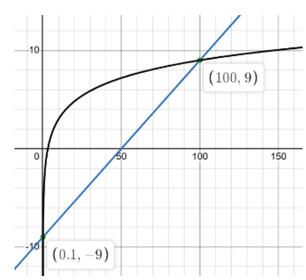
3. 
   a. Domain: $(3, \infty)$ Range: $(-\infty, \infty)$
   b. Domain: $(-\infty, \infty)$ Range: $(3, \infty)$
   c. Increasing (Rate of change of $g(x)$ is increasing since $g(x)$ is concave up)
   d. Negative (Rate of change of $g(x)$ is negative since $g(x)$ is decreasing)
   e. $\lim\limits_{x \to \infty} g(x) = -\infty$



4. 
   a. Domain: $(-\infty, 0)$ Range: $(-\infty, \infty)$
   b. Domain: $(-\infty, \infty)$ Range: $(-\infty, 0)$
   c. $\lim\limits_{x \to -\infty} h(x) = \infty$
   d. (D)



5. $f(x)$ B
   $g(x)$ D
   $h(x)$ A
   $k(x)$ C
6. (C)
7. $g(x)$ is always concave down $\Rightarrow$ $g(x)$ has no points of inflection.
8. 
   A.
      i. $a + b\log(0.1) = -9$
         $a + b\log(100) = 9$
      ii. $a = -3$ and $b = 6$
   B.
      i. $\frac{h(100)-h(0.1)}{100-0.1} = \frac{(9)-(-9)}{100-\frac{1}{10}} = 0.18$
      ii. $\frac{9-h(10)}{100-10} \approx 0.18018.\ h(10) \approx -7.216$
      iii. We are comparing the outputs of the concave down function $h$ to outputs of the secant line that passes through $h$ at $x = 0.1$ and $x = 100$. Being concave down, $h$ is above the secant line between $x = \frac{1}{10}$ and $x = 100$. So, the secant line underapproximates the actual value of $h(10)$.




9. $f(x)$ is always increasing $\Rightarrow$ $f(x)$ has no local maximum.
10. 
   a. Increasing
   b. Concave down
   c. 193
11. $f(x)$ is a concave down function. Therefore, the rate of change is decreasing. The average rate of change over $[3,6]$ is less than the average rate of change over $[0,3]$.

12.
   a. $9$
   b. $\frac{1}{3}$
   c. $k = 1$ (since $\log_3(3x) = \log_3 3 + \log_3 x = 1 + \log_3 x$)
   d. $c = \frac{1}{2}$ (since $g\left(\sqrt{x}\right) = \log_3 \sqrt{x} = \frac{1}{2}\log_3 x$)
13. D
14. $\log_3(x^2)$